(6)

(I) 20 units in S(A) requires 20 of S(A) and 50 from S(I)

(II) 10 units in S(I) requires 50 of S(A), 50 of S(I) and 80 of S(S)

(III) 10 units in S(S) requires 80 of S(A) and 20 from S(S)

(I) 1 unit in A → 0.1 of S(A) and 0.25 of S(I)

(II) 1 unit in I → 0.5 of S(A), 0.5 of S(I) and 0.8 of S(S)

(III) 1 unit in S → 0.8 of S(A) and 0.2 from S(S)

- Demand of x_a: $0.1 \cdot x_a + 0.5 \cdot x_I + 0.8 \cdot x_S + 20$
  (to satisfy production in other sectors; to satisfy final demand)
- Supply of x_a: x_a

$$x_a = 0.1\,x_a + 0.5\,x_I + 0.8\,x_S + 20$$

- Isolating final demand, we get:

$$0.9\,x_a - 0.5\,x_I - 0.8\,x_S = 20$$

- Similarly for x_I and x_S, we get:

Sector I: $x_I = 0.25\,x_a + 0.5 \cdot x_I$

$$-0.25\,x_a + 0.5\,x_I + 0 \cdot x_S = 0$$

Sector S: $x_S = 0.8\,x_I + 0.2\,x_S + 20$

$$0 \cdot x_a - 0.8\,x_I + 0.8\,x_S = 20$$

a)
$$\begin{pmatrix} 0.9 & -0.5 & -0.8 \\ -0.25 & 0.5 & 0 \\ 0 & -0.8 & +0.8 \end{pmatrix} \begin{pmatrix} x_A \\ x_I \\ x_S \end{pmatrix} = \begin{pmatrix} 20 \\ 0 \\ 20 \end{pmatrix}$$

b)
$$\det: \begin{vmatrix} 0.9 & -0.5 & -0.8 & 0.9 & -0.5 \\ -0.25 & 0.5 & 0 & -0.25 & 0.5 \\ 0 & -0.8 & 0.8 & 0 & -0.8 \end{vmatrix}$$

$= 0.9 \cdot 0.5 \cdot 0.8 - 0.8 \cdot 0.8 \cdot 0.5 - 0.8 \cdot 0.5 \cdot 0.25 =$

$= 0.36 - 0.16 - 0.1 = \boxed{0.4}$