

⑤

$$Y = C + I - S \Rightarrow 0 = (-1) \cdot C + (+1) \cdot Y + (-1) \cdot I + (+1) \cdot S$$

$$C = 0.75Y + 80 \Rightarrow 80 = (1) \cdot C + (-0.75) \cdot Y + (0) \cdot I + (0) \cdot S$$

$$I = \alpha Y + 20 \Rightarrow 20 = (0) \cdot C + (+\alpha) \cdot Y + (1) \cdot I + (0) \cdot S$$

$$S = 0.2(C + I) - 24 \Rightarrow 24 = (0.2) \cdot C + (0) \cdot Y + (0.2) \cdot I + (-1) \cdot S$$

a) • Write it in matrix form:

$$= A \begin{pmatrix} 1 & -1 & -1 & 1 \\ -0.75 & 1 & 0 & 0 \\ -\alpha & 0 & 1 & 0 \\ 0 & 0.2 & 0.2 & -1 \end{pmatrix} \cdot \begin{pmatrix} Y \\ C \\ I \\ S \end{pmatrix} = \begin{pmatrix} 0 \\ 80 \\ 20 \\ 24 \end{pmatrix}$$

• For what value of α is there not a unique eq.?

→ Remember: Inverse only exists iff $|A| \neq 0$.

* Use expansion of cofactors of a row to find the determinant

$$|A| =$$

i) Use row 2 as there are two 0's: (easier)

$$a) M_{21} = \begin{vmatrix} -1 & -1 & 1 \\ 0 & 1 & 0 \\ 0.2 & 0.2 & -1 \end{vmatrix} \rightarrow \begin{vmatrix} -1 & -1 & 1 & -1 & -1 \\ 0 & 1 & 0 & 0 & 1 \\ 0.2 & 0.2 & -1 & 0.2 & 0.2 \end{vmatrix}$$

$$\det \rightarrow (-1) \cdot (1) \cdot (-1) + (-1) \cdot (0) \cdot (0.2) + (1) \cdot (0) \cdot (0.2) - [0.2 \cdot (1) \cdot (1) + (0.2) \cdot (0) \cdot (-1) + (-1) \cdot (0) \cdot (-1)] =$$

$$\det \rightarrow (1) + (0) + (0) - [0.2 + 0 + 0] = 1 - 0.2 = \boxed{0.8}$$

$$b) M_{22} = \begin{vmatrix} 1 & -1 & 1 \\ -\alpha & 1 & 0 \\ 0 & 0.2 & -1 \end{vmatrix} \rightarrow \begin{vmatrix} 1 & -1 & 1 & -1 & -1 \\ -\alpha & 1 & 0 & -\alpha & 1 \\ 0 & 0.2 & -1 & 0 & 0.2 \end{vmatrix}$$

$$\det \rightarrow (1) \cdot (1) \cdot (-1) + (-1) \cdot (0) \cdot (0) + (1) \cdot (-\alpha) \cdot (0.2) - [0 \cdot (-1) \cdot (1) + (0.2) \cdot (0) \cdot (1) + (-1) \cdot (-\alpha) \cdot (-1)] =$$

$$= (-1) + (0) + (-0.2\alpha) - [0 + 0 + (-\alpha)] =$$

$$= -1 - 0.2\alpha + \alpha \Rightarrow -1 + (1 - 0.2)\alpha = \boxed{-1 + 0.8\alpha}$$

$$\det A = (-1)^{2+1} \cdot M_{21} + (-1)^{2+2} \cdot M_{22} = (-1)^3 (0.8) + (-1)^4 (-1 + 0.8\alpha) =$$

$$= -0.8 - 1 + 0.8\alpha \Rightarrow$$

$$(-0.75)(-1)^{2+1} \cdot (0.8) + (1) \cdot (-1)^{2+2} \cdot (-1 + 0.8\alpha) =$$

$$= (-0.75)(-1) \cdot (0.8) + (1) \cdot (1) \cdot (-1 + 0.8\alpha) = \frac{3}{4} \cdot \frac{4}{5} + -1 + 0.8\alpha =$$

$$= \frac{3}{5} - 1 + 0.8\alpha = \frac{-2}{5} + 0.8\alpha \Rightarrow \boxed{-0.4 + 0.8\alpha} = 0 \Rightarrow 0.8\alpha = 0.4$$

$$\alpha = \frac{1}{2}$$