9 Find maximum and minimum of

a) $f(x) = \dfrac{x^2+1}{x}$ on $[1/2, 2]$

- Step 1: Compute $f'(x) = 0$

→ By $\left(\dfrac{f(x)}{g(x)}\right)' = \dfrac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$, we get:

$$\frac{(2x)(x) - (x^2+1)(1)}{x \cdot x} = \frac{2x^2 - x^2 - 1}{x^2} = \boxed{\frac{x^2-1}{x^2} = 0}$$

Hence,

$$x^2 - 1 = 0 \Rightarrow x^2 = 1 \Rightarrow x = \pm\sqrt{1} \Rightarrow \boxed{x = \pm 1}$$

Only $x = +1$ is on our interval.

- Step 2: Compute $f''(x)$

→ Again, by the Quotient rule, we get:

$$f''(x) = \frac{(2x)(x^2) - (x^2-1)(2x)}{(x^2)^2} = \frac{(2x)(x^2) - (2x)(x^2) + 2x}{x^4} \Rightarrow$$

⇒ eliminating common elements in the denominator, we get:

$$f''(x) = \frac{2x}{x^4} = \boxed{2 \cdot \frac{1}{x^3}}$$; which, on $[1/2, 2]$ is > 0.

Hence, $x = 1$ is a minimum.

- Step 3: plug the bounds of the intervals to find the maximum

$$f(x=1/2) = \frac{(1/2)^2+1}{1/2} = \frac{1/4+1}{1/2} = \frac{\frac{5}{4}}{\frac{1}{2}} = \frac{10}{4} = 5/2$$ | As $f(x=1/2) = f(x=2)$,