(8) $f(x) = \frac{1}{\sqrt{2x-1}}$. (12)

Is $f(x)$ concave or convex on its domain?

- Firstly, let's calculate the domain of $f(x)$.

  - we know that $\frac{a}{b}$ is not well defined when $b=0$

  - We know that $\sqrt{2x-1}$ is not defined in the real numbers when $2x-1 \leq 0$

hence, to find the domain of $\frac{1}{\sqrt{2x-1}}$, we just need to find

$2x-1 < 0$, and exclude those numbers from the domain.

$\to 2x-1<0 \Rightarrow 2x<1 \Rightarrow \boxed{x<1/2}$

Hence, when $x<1/2$, $2(x)<1 \Rightarrow 2x-1<0 \Rightarrow \sqrt{2x-1} \nexists$.
Also, when $x=1/2$, $2(x)=2(1/2)=1 \Rightarrow 2x-1=0 \Rightarrow \sqrt{2x-1}=\sqrt{0}=0$.
Hence, $\sqrt{2x-1}$ when $x=1/2$ is 0 and, hence, $\frac{1}{\sqrt{2x-1}}$ is not well defined

Thus, the domain of $f(x)$ is $D_f = (1/2, \infty)$

- Now, let's find the second derivative in order to test for concavity / convexity.

Step 1: $f'(x) = (-1/2).(2x-1)^{-3/2} \cdot 2$

Step 2: $f''(x) = (-1/2)(-3/2).(2x-1)^{-5/2} \cdot 2 \cdot 2$

As $(-1/2)(-3/2) \cdot 2 \cdot 2 = \left(\frac{3}{4}\right) \cdot 4 = 3$, then:

Hence, $f''(x)>0$ in D_f and the function is convex

$f''(x) = 3 \cdot (2x-1)^{-5/2}$

Step 3: plug $x=1/2$ to get: $f''(x=1/2) = 3(2(1/2)-1)^{-5/2} = 0$.
As $x>1/2 \Rightarrow 3 \cdot (2(>1/2)-1)^{-5/2} = 3.(>0)^{-5/2} > 0$