④

$$A_t = \begin{pmatrix} 1 & 0 & t \\ 2 & 1 & t \\ 0 & -1 & 2 \end{pmatrix}$$

a) Determinant.

$$\left|\begin{matrix} 1 & 0 & t \\ 2 & 1 & t \\ 0 & -1 & 2 \end{matrix}\right| \begin{matrix} 1 & 0 \\ 2 & 1 \\ 0 & -1 \end{matrix}$$

$= (1)\cdot(1)\cdot(2) + (0)\cdot(t)\cdot(0) + (t)\cdot(2)(-1)$
$- [(0)\cdot(1)\cdot(t) + (-1)\cdot(t)\cdot(1) + (2)\cdot(2)\cdot(0)]$
$= 2 + 0 - 2t - 0 - (-t) - 0 =$
$= 2 - 2t + t = \boxed{2 - t}$

$|A_t^3|$

- By matrix rule: $A_t^n = A_t^{n-1} \cdot A_t$.
  $A_t^3 = A_t^2 \cdot A_t = A_t \cdot A_t \cdot A_t$.
- So,
  $|A_t^3| = |A_t \cdot A_t \cdot A_t|$.
- By matrix rule $|AB| = |A| \cdot |B|$,
  $|A_t \cdot A_t \cdot A_t| = |A_t A_t| \cdot |A_t| = \boxed{|A_t| \cdot |A_t| \cdot |A_t| = |A_t^3|}$
  $|A_{t=1}| = 2 - 1 = 1 \rightarrow |A_{t=1}^3| = |A_{t=1}| \cdot |A_{t=1}| \cdot |A_{t=1}| =$
  $= 1 \cdot 1 \cdot 1 = 1^3 = \boxed{1 = |A_{t=1}^3|}$

b) For which values of t does A_t have an inverse?
An inverse exists iff $|A_t| \neq 0$.
- So, $|A_t| = 0 \Rightarrow 2 - t = 0 \Rightarrow \boxed{t = 2}$
  $\boxed{\rightarrow |A_t| \neq 0 \text{ iff } t \neq 2}$

Inverse if $t = 0$

$$A_{t=0} = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & -1 & 2 \end{pmatrix}$$

1) $|A_{t=0}| = 2 - 0 = 2$

2) cofactor matrix:

$M_{11} = \begin{vmatrix} 1 & 0 \\ -1 & 2 \end{vmatrix} = 2 \rightarrow \boxed{C_{11} = (-1)^2 (2) = 2}$

$M_{12} = \begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix} = 4 \rightarrow \boxed{C_{12} = (-1)^3 (4) = -4}$

$M_{13} = \begin{vmatrix} 2 & 1 \\ 0 & -1 \end{vmatrix} = -2 \rightarrow \boxed{C_{13} = (-1)^4 (-2) = -2}$

$M_{21} = \begin{vmatrix} 0 & 0 \\ -1 & 2 \end{vmatrix} = 0 \rightarrow \boxed{C_{21} = 0}$

$M_{22} = \begin{vmatrix} 1 & 0 \\ 0 & 2 \end{vmatrix} = 2 \rightarrow \boxed{C_{22} = (-1)^4 (2) = 2}$

$M_{23} = \begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix} = -1 \rightarrow \boxed{C_{23} = (-1)^5 (-1) = 1}$

$M_{31} = \begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix} = 0 \rightarrow \boxed{C_{31} = 0}$

$M_{32} = \begin{vmatrix} 1 & 0 \\ 2 & 0 \end{vmatrix} = 0 \rightarrow C_{32} = 0$

$M_{33} = \begin{vmatrix} 1 & 0 \\ 2 & 1 \end{vmatrix} = 1 \rightarrow C_{33} = 1$