11 $c(q) = q^3 - 90q^2 + 7500q$

a) Find average and marginal cost:

- Avg. Cost $= \frac{c(q)}{q} = q^2 - 90q + 7500$

- Marginal Cost $= c'(q) = 3q^2 - 180q + 7500$

b) Find where the avg. cost is minimized.

- This just asks ask to compute the first derivative and equal it to 0:

$$(q^2 - 90q + 7500)' = 0 \Rightarrow 2q - 90 = 0 \Rightarrow \boxed{q = \frac{90}{2} = 45}$$

- Compare the values of average and marginal costs at that point.

i) $\frac{c(q=45)}{45} \Rightarrow (45)^2 - 90(45) + 7500 = 2{,}025 - 4{,}050 + 7500$

$\rightarrow$ Avg Cost $(q=45) = \boxed{5{,}475}$

ii) $c'(q=45) \Rightarrow 3(45)^2 - (180)(45) + 7500 = 6{,}075 - 8{,}100 + 7500 \Rightarrow$

$\rightarrow c'(q=45) = \boxed{5{,}475}$

* Extra Question: why does this happen?

$$\frac{\partial\left(\frac{c(q)}{q}\right)}{\partial q} = 0 \Rightarrow \frac{c'(q)\,q - c(q)\cdot q}{q \cdot q} \Rightarrow$$

$\Rightarrow \rightarrow \frac{a-b}{c} = \frac{a}{c} - \frac{b}{c}$. Hence, $\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a \cdot d}{b \cdot c} \rightarrow \frac{\frac{c(q)}{1}}{\frac{q \cdot q}{q}} =$

$\frac{c'(q)\,q}{q \cdot q} - \frac{c(q)}{q \cdot q} \rightarrow$

$= \boxed{\frac{c(q)}{q}}$