

(1) • Firm sells Q_A

• Demand function: $Q_A = 3000 - 3P_A$

• Step 1: Inverse demand function:

$$3P_A = 3000 - Q_A$$

$$P_A = 1000 - \frac{1}{3} Q_A$$

• Firm buys $Q_B = Q_A$

• Supply of market B: $Q_B = 6P_B - 4800$

• Step 2: Inverse demand function

$$\frac{Q_B + 4800}{6} = P_B \Rightarrow P_B = \left(\frac{1}{6}\right) Q_B + 800$$

• Transport cost from market A to market B: $100 Q$

a) Profit of the firm: three elements.

Enters positively ① the revenues he gets from selling in market A: $P_A \cdot Q_A$

Enters negatively ② The cost he faces when buying in market B: $P_B \cdot Q_B$

Enters negatively ③ The transportation costs: $100 \cdot Q$

$$\pi_i = P_A \cdot Q_A - P_B \cdot Q_B - 100 \cdot Q$$

Step 3: In eq, $Q_A = Q_B = Q$

Step 4: Substitute P_A and P_B in π_i :

$$\pi_i = \left(1000 - \left(\frac{1}{3}\right) Q_A\right) \cdot Q_A - \left(\left(\frac{1}{6}\right) Q_B + 800\right) Q_B - 100 Q$$

$$\Rightarrow \pi_i = 1000 Q - \frac{1}{3} Q^2 - \frac{1}{6} Q^2 + 800 Q - 100 Q$$

$$\Rightarrow \pi_i = 100 Q - \frac{1}{2} Q^2$$

b) Maximize profit (Q is our variable)

$$\max_Q \pi_i = 100 Q - \frac{1}{2} Q^2$$

$$\rightarrow \frac{\partial \pi_i}{\partial Q} = 0 \Rightarrow 100 - Q = 0 \Rightarrow Q = 100$$

$$\frac{\partial^2 \pi_i}{\partial Q^2} = -1 < 0 \rightarrow \text{maximum}$$

c) Now, firm profits face a new cost: the per unit tax: $t \cdot Q$.
 $\pi_i'(Q) = 0 \Rightarrow 100 - Q - t = 0 \Rightarrow Q = 100 - t$

$$\pi_i = 100 Q - \frac{1}{2} Q^2 - t Q$$

$$d) \text{ tax revenue: } t \cdot Q^* = t \cdot (100 - t) = 100t - t^2$$

$$\begin{aligned} \pi_i'(Q) &= 0 \Rightarrow 100 - 2t = 0 \\ \pi_i''(t) &= -2 < 0 \Rightarrow 50 = t \end{aligned}$$