• Step 5: Using the rule $\frac{a-b}{c} = \frac{a}{c} - \frac{b}{c}$, we can rewrite the previous expression as:

$$\frac{F'(x)}{F(x)} = \frac{f'(x) \cdot g(x)}{f(x) \cdot g(x)} - \frac{f(x) \cdot g'(x)}{f(x) \cdot g(x)}.$$

• Step 6: Eliminating the common elements in the numerator and denominator, we get:

$$\boxed{\frac{F'(x)}{F(x)} = \frac{f'(x)}{f(x)} - \frac{g'(x)}{g(x)}}$$

which is what we wanted to find.

Now, let's prove ii)

• The formula of the elasticity is $\varepsilon_{F,x} = \frac{\partial F(x)}{\partial x} \cdot \frac{x}{F(x)}$

• From the previous part, we know $\frac{\partial F(x)}{\partial x} = F'(x) = \frac{f'(x) g(x) - f(x) g'(x)}{g(x) \cdot g(x)}$.

• Hence, $\frac{F'(x) \cdot x}{F(x)} = \frac{F'(x)}{F(x)} \cdot x$.

• As $\frac{F'(x)}{F(x)} = \frac{f'(x)}{f(x)} - \frac{g'(x)}{g(x)}$ from the previous exercise, we can rewrite $\varepsilon_{F,x} = \frac{F'(x) \cdot x}{F(x)}$ as:

$$\varepsilon_{F,x} = \left[\frac{f'(x)}{f(x)} - \frac{g'(x)}{g(x)}\right] \cdot x = \frac{f'(x)}{f(x)} \cdot x - \frac{g'(x)}{g(x)} \cdot x.$$

• Also, we know that $\varepsilon_{f,x} = f'(x) \cdot \frac{x}{f(x)} = \frac{f'(x)}{f(x)} \cdot x$. Hence,