

(a) b)  $f(x) = x^2 \cdot e^{-x}$  for  $x \geq 0$

•  $f'(x) = 0 \Rightarrow$  As  $(f(x) \cdot g(x))' = f'(x) \cdot g(x) + f(x) \cdot g'(x)$ ,  
we get:

$$f'(x) = 0 \Rightarrow 2x \cdot e^{-x} - (1) \cdot x^2 \cdot e^{-x} = 0$$

$$\Rightarrow e^{-x} \cdot (2x - x^2) = 0$$

$$\Rightarrow e^{-x} \cdot x(2-x) = 0$$

$$\Rightarrow \text{As } x^{-a} = \frac{1}{x^a}, \text{ we get:}$$

$$f'(x) = \frac{x(2-x)}{e^x} = 0; \text{ which is 0 whenever the } \underline{\text{denominator}}$$

numerator is 0. Hence,

$$\boxed{x=0} \text{ or } 2-x=0 \Rightarrow \boxed{x=2}$$

$$x(2-x)=0 \Rightarrow$$

• Now, check  $f'(0)$  and  $f'(2)$ :

$$\rightarrow f'(x=0) = 0$$

$$\rightarrow f'(x=2) = \frac{2(2-2)}{e^2} = 0$$

• Now, check  $f'(0 < x < 2)$  and  $f'(x > 2)$ :

$$\rightarrow f'(x=1) = \frac{1(2-1)}{e^1} = \frac{1}{e} = \boxed{e^{-1} > 0}$$

$$\rightarrow f'(x=3) = \frac{3(2-3)}{e^3} = \frac{1}{e^3} \cdot (3(-1)) \Rightarrow \frac{1}{e^3} (-3) = \boxed{-3 \cdot \frac{1}{e^3} < 0}$$

Hence, as  $f'$  is increasing between 0 and 2 and decreasing after wards, 0 is the minimum and 2 the maximum.