6 prove that, if $F(x) = \frac{f(x)}{g(x)}$, then

i) $\frac{F'(x)}{F(x)} = \frac{f'(x)}{f(x)} - \frac{g'(x)}{g(x)}$

ii) $\varepsilon_{F,x} = \varepsilon_{f,x} - \varepsilon_{g,x}$

Let's start with i).

- Step 1: Use the rule $\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{(g(x))^2}$, to get:

$$F'(x) = \left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{g(x) \cdot g(x)}$$

- Step 2: Provide the formula for $\frac{F'(x)}{F(x)}$.

$$\frac{F'(x)}{F(x)} = \frac{F'(x)}{\frac{f(x)}{g(x)}} = \frac{\frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{g(x) \cdot g(x)}}{\frac{f(x)}{g(x)}}$$

- Step 3: Use the rule $\frac{\frac{a}{b}}{\frac{c}{d}}$, ~~drg~~ $= \frac{a \cdot d}{b \cdot c}$, to get:

$$\frac{F'(x)}{F(x)} = \frac{[f'(x) \cdot g(x) - f(x) \cdot g'(x)] \cdot g(x)}{g(x) \cdot g(x) \cdot f(x)}$$

- Step 4: Eliminating common elements of numerator and denominator, we get:

$$\frac{F'(x)}{F(x)} = \frac{f'(x)\, g(x) - f(x) \cdot g'(x)}{f(x) \cdot g(x)}$$