5 $c(q) = 20 - 4(25 - \frac{1}{2}q)^{1/2}$. Find the marginal cost when $q = 32$

- Finding the marginal cost just means computing the first derivative.

- We basically have to apply the chain rule here.

Note that, assuming $g(q) = (25 - \frac{1}{2}q)$

and assuming $f(q) = 20 - 4(q)^{1/2}$.

then $f(g(q)) = f \circ g(q) = 20 - 4(g(q))^{1/2} = 20 - 4(25 - 1/2\, q)^{1/2} = c(q)$

The chain rule says: $(f(g(x)))' = f'(g(x)) \cdot g'(x)$.

- $f'(q) = -4(1/2) \cdot (q)^{-1/2} \rightarrow f'(g(q)) = (-4)(1/2) \cdot (25 - 1/2\, q)^{-1/2}$

- $g'(q) = -1/2$

Hence, $f'(g(q)) \cdot g'(q) = (-4)(1/2) \cdot (25 - 1/2\, q)^{-1/2} \cdot (-1/2)$

- Noting that $(-4)(1/2) \cdot (-1/2) = (-4)(-1/4) = \frac{-4}{-4} = +1$, we can rewrite the previous expression as:

$$c'(q) = +(25 - 1/2\, q)^{-1/2}$$

- Finally, finding the marginal cost when $q = 32$ just asks for computing $c'(q = 32)$. Hence,

$$c'(q=32) = (25 - (1/2)(32))^{-1/2} = (25 - 16)^{-1/2} =$$

$$\Rightarrow c'(q=32) = (9)^{-1/2} = \frac{1}{\sqrt{9}} = \boxed{\frac{1}{3} = c'(q=32)}$$