$$D(Q) = b - a(q_1 + q_2)$$

$$TC(q) = c_i q^2$$

Cournot:

$$\pi_1(q_1, q_2) = \big(b - a(q_1 + q_2)\big) \times q_1 - c_1 q_1^2$$

$$\pi_1(q_1, q_2) = bq_1 - aq_1^2 - aq_1q_2 - c_1q_1^2$$

- Firm 1 maximises income:

$$\frac{\partial \pi_1(q_1, q_2)}{\partial q_1} = 0 \rightarrow b - 2aq_1 - aq_2 - 2c_1q_1 = 0$$

Isolating q_1 in the RHS, we get:

$$b - aq_2 = 2aq_1 + 2c_1q_1$$

Taking q_1 as a common factor in the RHS, we can rewrite the previous expression as:

$$b - aq_2 = q_1 \times (2a + 2c_1)$$

Multiplying both hand sides by $\frac{1}{(2a+2c_1)}$, we get:

$$\frac{b - aq_2}{2a + 2c_1} = q_1$$

Taking 2 as a common factor in the denominator of the LHS, we get:

$$\frac{b - aq_2}{2 \times (a + c_1)} = q_1$$

Firm 2's income is defined by:

$$\pi_2(q_1, q_2) = \big(b - a(q_1 + q_2)\big) \times q_2 - c_2 q_2^2$$

$$\pi_2(q_1, q_2) = bq_2 - aq_1q_2 - aq_2^2 - c_2q_2^2$$

- Firm 2 maximises income:

$$\frac{\partial \pi_2(q_1, q_2)}{\partial q_1} = 0 \rightarrow b - aq_1 - 2aq_2 - 2c_2q_2 = 0$$

Isolating q_1 in the RHS, we get:

$$b - aq_1 = 2aq_2 + 2c_2q_2$$

Taking q_1 as a common factor in the RHS, we can rewrite the previous expression as:

$$b - aq_1 = q_2 \times (2a + 2c_2)$$

Multiplying both hand sides by $\frac{1}{(2a+2c_2)}$, we get:

$$\frac{b - aq_1}{2a + 2c_2} = q_2$$

Taking 2 as a common factor in the denominator of the LHS, we get:

$$\frac{b - aq_1}{2 \times (a + c_2)} = q_2$$

Substitute q_2 into q_1 to get:

$$q_1 = \frac{b - a\left(\frac{b - aq_1}{2 \times (a + c_2)}\right)}{2 \times (a + c_1)}$$

Expanding the parenthesis in the numerator, we get:

$$q_1 = \frac{b - \frac{ab - a^2 q_1}{2 \times (a + c_2)}}{2 \times (a + c_1)}$$

Multiplying b by $\frac{2\times(a+c_2)}{2\times(a+c_2)} = 1$, we get:

$$q_1 = \frac{\frac{2 \times (a + c_2) \times b - ab + a^2 q_1}{2 \times (a + c_2)}}{2 \times (a + c_1)}$$

Noting that $\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{ad}{bc}$, we can rewrite the previous expression as:

$$q_1 = \frac{2 \times (a + c_2) \times b - ab + a^2 q_1}{2 \times (a + c_2) \times 2 \times (a + c_1)}$$

Bringing the denominator of the RHS to the LHS, we get:

$$q_1 \times 2 \times (a + c_2) \times 2 \times (a + c_1) = 2 \times (a + c_2) \times b - ab + a^2 q_1$$

Simplifying the LHS, we get:

$$q_1 \times 4 \times (a + c_2) \times (a + c_1) = 2 \times (a + c_2) \times b - ab + a^2 q_1$$

Expanding the parentheses of the LHS, we get:

$$q_1 \times 4 \times (a^2 + ac_1 + ac_2 + c_1 c_2) = 2 \times (a + c_2) \times b - ab + a^2 q_1$$

Again, expanding the parenthesis of the LHS, we get:

$$q_1 \times (4a^2 + 4ac_1 + 4ac_2 + 4c_1 c_2) = 2 \times (a + c_2) \times b - ab + a^2 q_1$$

Isolating q_1 in the LHS, we get:

$$q_1 \times (4a^2 + 4ac_1 + 4ac_2 + 4c_1c_2) - q_1a^2 = 2 \times (a + c_2) \times b - ab$$

Taking q_1 as a common factor in the LHS, we get:

$$q_1 \times (4a^2 - a^2 + 4ac_1 + 4ac_2 + 4c_1c_2) = 2 \times (a + c_2) \times b - ab$$

Expanding the parenthesis of the RHS, we get:

$$q_1 \times (4a^2 - a^2 + 4ac_1 + 4ac_2 + 4c_1c_2) = 2ab + 2bc_2 - ab$$

Simplifying, we get:

$$q_1 \times (3a^2 + 4ac_1 + 4ac_2 + 4c_1c_2) = ab + 2bc_2$$

Taking b as a common factor in the RHS, we get:

$$q_1 \times (3a^2 + 4ac_1 + 4ac_2 + 4c_1c_2) = b \times (a + 2c_2)$$

Dividing both hand sides by $(3a^2 + 4ac_1 + 4ac_2 + 4c_1c_2)$, we get:

$$q_1 = \frac{b \times (a + 2c_2)}{(3a^2 + 4ac_1 + 4ac_2 + 4c_1c_2)}$$

Substituting the value of q_1 into the expression of q_2 found earlier, we get:

$$\frac{b - a \times \left(\frac{b \times (a + 2c_2)}{(3a^2 + 4ac_1 + 4ac_2 + 4c_1c_2)}\right)}{2 \times (a + c_2)} = q_2$$

Multiplying b by $\frac{3a^2+4ac_1+4ac_2+4c_1c_2}{3a^2+4ac_1+4ac_2+4c_1c_2} = 1$, we get:

$$\frac{\left(\frac{b \times (3a^2 + 4ac_1 + 4ac_2 + 4c_1c_2) - ab \times (a + 2c_2)}{(3a^2 + 4ac_1 + 4ac_2 + 4c_1c_2)}\right)}{2 \times (a + c_2)} = q_2$$

Which can be rewritten as (we are only multiplying one a by every element within the parenthesis it was multiplying):

$$\frac{\left(\frac{b \times (3a^2 + 4ac_1 + 4ac_2 + 4c_1c_2) - b \times (a^2 + 2ac_2)}{(3a^2 + 4ac_1 + 4ac_2 + 4c_1c_2)}\right)}{2 \times (a + c_2)} = q_2$$

Taking b as a common factor in the numerator, we get:

$$\frac{\left(\frac{b \times (3a^2 + 4ac_1 + 4ac_2 + 4c_1c_2 - a^2 - 2ac_2)}{(3a^2 + 4ac_1 + 4ac_2 + 4c_1c_2)}\right)}{2 \times (a + c_2)} = q_2$$

Simplifying, we get:

$$\frac{\left(\frac{b \times (2a^2 + 4ac_1 + 2ac_2 + 4c_1c_2)}{(3a^2 + 4ac_1 + 4ac_2 + 4c_1c_2)}\right)}{2 \times (a + c_2)} = q_2$$

Taking $4c_1$ and $2a$ as a common factor in the numerator, we get:

$$\frac{\left(\frac{b \times \left(2a \times (a + c_2) + 4c_1 \times (a + c_2)\right)}{(3a^2 + 4ac_1 + 4ac_2 + 4c_1c_2)}\right)}{2 \times (a + c_2)} = q_2$$

Taking $a + c_2$ as a common factor in the numerator, we get:

$$\frac{\left(\frac{b \times \left((a + c_2) \times (2a + 4c_1)\right)}{(3a^2 + 4ac_1 + 4ac_2 + 4c_1c_2)}\right)}{2 \times (a + c_2)} = q_2$$

Taking 2 as a common factor in $2a + 4c_1$, we can rewrite the previous expression as:

$$\frac{\left(\frac{b \times (2 \times (a + c_2) \times (a + 2c_1))}{(3a^2 + 4ac_1 + 4ac_2 + 4c_1c_2)}\right)}{2 \times (a + c_2)} = q_2$$

Again, noting that $\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{ad}{bc}$, we can rewrite the previous expression as:

$$\left(\frac{b \times (2 \times (a + c_2) \times (a + 2c_1))}{(3a^2 + 4ac_1 + 4ac_2 + 4c_1c_2) \times 2 \times (a + c_2)}\right) = q_2$$

And, simplifying, we get our final expression:

$$\frac{b \times (a + 2c_1)}{(3a^2 + 4ac_1 + 4ac_2 + 4c_1c_2)} = q_2$$

To find the price, substitute the expressions we found for q_1 and q_2 into $D(Q)$ to get:

$$D(Q) = b - a\left(\frac{b \times (a + 2c_2)}{(3a^2 + 4ac_1 + 4ac_2 + 4c_1c_2)} + \frac{b \times (a + 2c_1)}{(3a^2 + 4ac_1 + 4ac_2 + 4c_1c_2)}\right)$$

Taking b as a common factor within the parenthesis, we get:

$$D(Q) = b - a\left(\frac{b \times (a + 2c_2 + a + 2c_1)}{(3a^2 + 4ac_1 + 4ac_2 + 4c_1c_2)}\right)$$

Which simplifies to:

$$D(Q) = b - \frac{2ab \times (a + c_2 + c_1)}{(3a^2 + 4ac_1 + 4ac_2 + 4c_1c_2)}$$

Which is equivalent to:

$$D(Q) = \frac{b \times (3a^2 + 4ac_1 + 4ac_2 + 4c_1c_2) - 2ab \times (a + c_2 + c_1)}{(3a^2 + 4ac_1 + 4ac_2 + 4c_1c_2)}$$

Expanding the parentheses in the numerator, we get:

$$D(Q) = \frac{3a^2b + 4abc_1 + 4abc_2 + 4bc_1c_2 - 2a^2b - 2abc_2 - 2abc_1}{(3a^2 + 4ac_1 + 4ac_2 + 4c_1c_2)}$$

Which, after simplifying, becomes:

$$D(Q) = \frac{a^2b + 2abc_1 + 2abc_2 + 4bc_1c_2}{(3a^2 + 4ac_1 + 4ac_2 + 4c_1c_2)}$$

Which, taking b as a common factor, becomes:

$$D(Q) = \frac{b \times (a^2 + 2ac_1 + 2ac_2 + 4c_1c_2)}{(3a^2 + 4ac_1 + 4ac_2 + 4c_1c_2)}$$

Taking $2c_1$ as a common factor in the numerator, we get:

$$D(Q) = \frac{b \times (a^2 + 2c_1 \times (a + 2c_2) + 2ac_2)}{(3a^2 + 4ac_1 + 4ac_2 + 4c_1c_2)}$$

Taking a as a common factor in the elements outside of the parenthesis in the numerator, we get:

$$D(Q) = \frac{b \times \left(a \times (a + 2c_2) + 2c_1 \times (a + 2c_2)\right)}{(3a^2 + 4ac_1 + 4ac_2 + 4c_1c_2)}$$

Taking $a + 2c_2$ as a common factor in the numerator, we get:

$$D(Q) = \frac{b \times \left((a + 2c_2) \times (a + 2c_1)\right)}{(3a^2 + 4ac_1 + 4ac_2 + 4c_1c_2)} = p^*$$

A different way to express the optimal price is in terms of q_1 and q_2.

Given that:

$$q_1 = \frac{b \times (a + 2c_2)}{(3a^2 + 4ac_1 + 4ac_2 + 4c_1c_2)}$$

And

$$\frac{b \times (a + 2c_1)}{(3a^2 + 4ac_1 + 4ac_2 + 4c_1c_2)} = q_2$$

Then, it trivially follows that:

$$(a + 2c_2) = q_1 \times \frac{(3a^2 + 4ac_1 + 4ac_2 + 4c_1c_2)}{b}$$

$$(a + 2c_1) = q_2 \times \frac{(3a^2 + 4ac_1 + 4ac_2 + 4c_1c_2)}{b}$$

Substituting those into p^*, we get:

$$p^* = \frac{b \times q_1 \times \frac{(3a^2 + 4ac_1 + 4ac_2 + 4c_1c_2)}{b} \times q_2 \times \frac{(3a^2 + 4ac_1 + 4ac_2 + 4c_1c_2)}{b}}{(3a^2 + 4ac_1 + 4ac_2 + 4c_1c_2)}$$

Which can be rewritten as:

$$p^* = \frac{\frac{b \times q_1 \times q_2 \times (3a^2 + 4ac_1 + 4ac_2 + 4c_1c_2)^2}{b^2}}{(3a^2 + 4ac_1 + 4ac_2 + 4c_1c_2)}$$

And again, noting that $\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{ad}{bc}$, we get:

$$p^* = \frac{b \times q_1 \times q_2 \times (3a^2 + 4ac_1 + 4ac_2 + 4c_1c_2)^2}{b^2 \times (3a^2 + 4ac_1 + 4ac_2 + 4c_1c_2)}$$

Which can be simplified to:

$$p^* = q_1 \times q_2 \times \frac{(3a^2 + 4ac_1 + 4ac_2 + 4c_1c_2)}{b}$$

Hence, the profit of both firms is driven by:

$$\pi_1(q_1, q_2) = p^* \times q_1^* - c_1 q_1^{*^2}$$

$$\pi_2(q_1, q_2) = p^* \times q_2^* - c_1 q_2^{*^2}$$

Which, after substituting p^*, q_1^* and q_2^*, the profit of firm 1 becomes:

$$\pi_1(q_1, q_2) = q_1^2 \times q_2 \times \frac{(3a^2 + 4ac_1 + 4ac_2 + 4c_1c_2)}{b} - c_1 q_1^2$$

$$\pi_1(q_1, q_2) = q_1^2 \times \left(q_2 \times \frac{(3a^2 + 4ac_1 + 4ac_2 + 4c_1c_2)}{b} - c_1 \right)$$

$$\pi_1(q_1, q_2) = q_1^2 \times \left(\frac{b \times (a + 2c_1)}{(3a^2 + 4ac_1 + 4ac_2 + 4c_1c_2)} \times \frac{(3a^2 + 4ac_1 + 4ac_2 + 4c_1c_2)}{b} - c_1 \right)$$

$$\pi_1(q_1, q_2) = q_1^2 \times (a + 2c_1 - c_1)$$

$$\pi_1(q_1, q_2) = q_1^2 \times (a + c_1)$$

Analogously, after substituting p^*, q_1^* and q_2^*, the profit of firm 1 becomes:

$$\pi_2(q_1, q_2) = q_1 \times q_2^2 \times \frac{(3a^2 + 4ac_1 + 4ac_2 + 4c_1c_2)}{b} - c_2 q_2^2$$

$$\pi_2(q_1, q_2) = q_2^2 \times \left(q_1 \times \frac{(3a^2 + 4ac_1 + 4ac_2 + 4c_1c_2)}{b} - c_2 \right)$$

$$\pi_2(q_1, q_2) = q_2^2 \times \left(\frac{b \times (a + 2c_2)}{(3a^2 + 4ac_1 + 4ac_2 + 4c_1c_2)} \times \frac{(3a^2 + 4ac_1 + 4ac_2 + 4c_1c_2)}{b} - c_2 \right)$$

$$\pi_2(q_1, q_2) = q_2^2 \times (a + 2c_2 - c_2)$$

$$\pi_2(q_1, q_2) = q_2^2 \times (a + c_2)$$

Stackelberg:

Firm 2 moves second, and hence we find the same q_2 as before:

$$\frac{b - aq_1}{2 \times (a + c_2)} = q_2$$

Firm 1 knows firm 2's optimal output, q_2. Hence, he substitutes it into his own profit function:

$$\pi_1(q_1, q_2^*) = bq_1 - aq_1^2 - aq_1 \times \frac{b - aq_1}{2 \times (a + c_2)} - c_1 q_1^2$$

Which becomes:

$$\pi_1(q_1, q_2^*) = bq_1 - aq_1^2 + \frac{a^2 q_1^2 - abq_1}{2 \times (a + c_2)} - c_1 q_1^2$$

- Firm 1 maximises income:

$$\frac{\partial \pi_1(q_1, q_2)}{\partial q_1} = 0 \rightarrow b - 2aq_1 + \frac{2a^2 q_1 - ab}{2 \times (a + c_2)} - 2c_1 q_1 = 0$$

Which is equivalent to:

$$b - 2aq_1 + \frac{2a^2 q_1}{2 \times (a + c_2)} - \frac{ab}{2 \times (a + c_2)} - 2c_1 q_1 = 0$$

Isolating q_1 in the RHS, we get:

$$b - \frac{ab}{2 \times (a + c_2)} = 2c_1 q_1 + 2aq_1 - \frac{2a^2 q_1}{2 \times (a + c_2)}$$

Taking q_1 as a common factor, we get:

$$b - \frac{ab}{2 \times (a + c_2)} = q_1 \times \left(2c_1 + 2a - \frac{2a^2}{2 \times (a + c_2)}\right)$$

Multiplying b, $2c_1$ and $2a$ by $\frac{2\times(a+c_2)}{2\times(a+c_2)} = 1$, we get:

$$\frac{2 \times (a + c_2) \times b - ab}{2 \times (a + c_2)} = q_1 \times \left(\frac{2c_1 \times 2 \times (a + c_2) + 2a \times 2 \times (a + c_2) - 2a^2}{2 \times (a + c_2)}\right)$$

Which is equivalent to:

$$2 \times (a + c_2) \times b - ab = q_1 \times (2c_1 \times 2 \times (a + c_2) + 2a \times 2 \times (a + c_2) - 2a^2)$$

Expanding the parentheses, we get:

$$2ab + 2bc_2 - ab = q_1 \times (4ac_1 + 4c_1c_2 + 4a^2 + 4ac_2 - 2a^2)$$

Which, simplifying, we get:

$$ab + 2bc_2 = q_1 \times (4ac_1 + 4c_1c_2 + 2a^2 + 4ac_2)$$

Isolating q_1 in the RHS, we get:

$$\frac{ab + 2bc_2}{4ac_1 + 4c_1c_2 + 2a^2 + 4ac_2} = q_1$$

Taking b as a common factor in the numerator and 2 as a common factor in the denominator, we get:

$$\frac{b \times (a + 2c_2)}{2 \times (a^2 + 2ac_1 + 2ac_2 + 2c_1c_2)} = q_1^*$$

Substituting q_1^* into q_2, we get:

$$\frac{b - a \times \left(\frac{b \times (a + 2c_2)}{2 \times (a^2 + 2ac_1 + 2ac_2 + 2c_1c_2)}\right)}{2 \times (a + c_2)} = q_2$$

Multiplying b by $\frac{2\times(a^2+2ac_1+2ac_2+2c_1c_2)}{2\times(a^2+2ac_1+2ac_2+2c_1c_2)} = 1$, and noting that $\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{ad}{bc}$, we get:

$$\frac{2b \times (a^2 + 2ac_1 + 2ac_2 + 2c_1c_2) - ab \times (a + 2c_2)}{2 \times (a^2 + 2ac_1 + 2ac_2 + 2c_1c_2) \times 2 \times (a + c_2)} = q_2$$

Which can be rewritten as:

$$\frac{b \times (2a^2 + 4ac_1 + 4ac_2 + 4c_1c_2) - b \times (a^2 + 2ac_2)}{2 \times (a^2 + 2ac_1 + 2ac_2 + 2c_1c_2) \times 2 \times (a + c_2)} = q_2$$

Taking b as a common factor in the numerator, we get:

$$\frac{b \times (2a^2 + 4ac_1 + 4ac_2 + 4c_1c_2 - a^2 - 2ac_2)}{2 \times (a^2 + 2ac_1 + 2ac_2 + 2c_1c_2) \times 2 \times (a + c_2)} = q_2$$

Which can be simplified to:

$$\frac{b \times (a^2 + 2ac_2 + 4ac_1 + 4c_1c_2)}{2 \times (a^2 + 2ac_1 + 2ac_2 + 2c_1c_2) \times 2 \times (a + c_2)} = q_2$$

And can be rewritten as:

$$\frac{b \times (a^2 + 2ac_2 + 4ac_1 + 4c_1c_2)}{4 \times (a^2 + 2ac_1 + 2ac_2 + 2c_1c_2) \times (a + c_2)} = q_2^*$$

Notice, also, that we can rewrite it in terms of q_1 if we wanted to. First, let's take a and $4c_1$ as common factors in the numerator to get:

$$\frac{b \times \left(a \times (a + 2c_2) + 4c_1 \times (a + c_2)\right)}{4 \times (a^2 + 2ac_1 + 2ac_2 + 2c_1c_2) \times (a + c_2)} = q_2$$

Then, let's split the fraction into two fractions:

$$\frac{ab \times (a + 2c_2)}{4 \times (a^2 + 2ac_1 + 2ac_2 + 2c_1c_2) \times (a + c_2)} + \frac{4bc_1 \times (a + c_2)}{4 \times (a^2 + 2ac_1 + 2ac_2 + 2c_1c_2) \times (a + c_2)} = q_2$$

Let's rewrite the first fraction to get:

$$\left(\frac{a}{2 \times (a + c_2)}\right) \times \frac{b \times (a + 2c_2)}{2 \times (a^2 + 2ac_1 + 2ac_2 + 2c_1c_2)} + \frac{4bc_1 \times (a + c_2)}{4 \times (a^2 + 2ac_1 + 2ac_2 + 2c_1c_2) \times (a + c_2)} = q_2$$

And, noticing that $\frac{b \times (a+2c_2)}{2 \times (a^2+2ac_1+2ac_2+2c_1c_2)} = q_1$, we can rewrite the previous expression as:

$$\left(\frac{a}{2 \times (a + c_2)}\right) \times q_1^* + \frac{4bc_1 \times (a + c_2)}{4 \times (a^2 + 2ac_1 + 2ac_2 + 2c_1c_2) \times (a + c_2)} = q_2^*$$

Now, to find the optimal price we substitute q_1^* and q_2^* into $D(Q)$ to get:

$$D(Q) = b - a\left(\frac{b \times (a + 2c_2)}{2 \times (a^2 + 2ac_1 + 2ac_2 + 2c_1c_2)} + \frac{b \times (a^2 + 2ac_2 + 4ac_1 + 4c_1c_2)}{4 \times (a^2 + 2ac_1 + 2ac_2 + 2c_1c_2) \times (a + c_2)}\right)$$

Multiplying the first element of the parenthesis by $\frac{2\times(a+c_2)}{2\times(a+c_2)} = 1$, we get:

$$D(Q) = b - a\left(\frac{2b \times (a + 2c_2) \times (a + c_2)}{4 \times (a^2 + 2ac_1 + 2ac_2 + 2c_1c_2) \times (a + c_2)} + \frac{b \times (a^2 + 2ac_2 + 4ac_1 + 4c_1c_2)}{4 \times (a^2 + 2ac_1 + 2ac_2 + 2c_1c_2) \times (a + c_2)}\right)$$

Taking b as a common factor in both fractions, we get:

$$D(Q) = b - a\left(\frac{b \times (2 \times (a + 2c_2) \times (a + c_2) + a^2 + 2ac_2 + 4ac_1 + 4c_1c_2)}{4 \times (a^2 + 2ac_1 + 2ac_2 + 2c_1c_2) \times (a + c_2)}\right)$$

Multiplying the first b by $\frac{4\times(a^2+2ac_1+2ac_2+2c_1c_2)\times(a+c_2)}{4\times(a^2+2ac_1+2ac_2+2c_1c_2)\times(a+c_2)} = 1$, we get:

$$D(Q) = \frac{b \times \left(4 \times (a^2 + 2ac_1 + 2ac_2 + 2c_1c_2) \times (a + c_2)\right)}{4 \times (a^2 + 2ac_1 + 2ac_2 + 2c_1c_2) \times (a + c_2)} - b\left(\frac{a \times (2 \times (a + 2c_2) \times (a + c_2) + a^2 + 2ac_2 + 4ac_1 + 4c_1c_2)}{4 \times (a^2 + 2ac_1 + 2ac_2 + 2c_1c_2) \times (a + c_2)}\right)$$

Which can be rewritten in a single fraction:

$$D(Q) = \frac{b \times \left(4 \times (a^2 + 2ac_1 + 2ac_2 + 2c_1c_2) \times (a + c_2) - a \times (2 \times (a + 2c_2) \times (a + c_2) + a^2 + 2ac_2 + 4ac_1 + 4c_1c_2)\right)}{4 \times (a^2 + 2ac_1 + 2ac_2 + 2c_1c_2) \times (a + c_2)}$$

We can separate the second term in the numerator by multiplying a by $2 \times (a + 2c_2) \times (a + c_2)$ and $(a^2 + 2ac_2 + 4ac_1 + 4c_1c_2)$ separately to get:

$$D(Q) = \frac{b \times \Big(4 \times (a^2 + 2ac_1 + 2ac_2 + 2c_1c_2) \times (a + c_2) - \big(2a \times (a + 2c_2) \times (a + c_2)\big) - a \times (a^2 + 2ac_2 + 4ac_1 + 4c_1c_2)\Big)}{4 \times (a^2 + 2ac_1 + 2ac_2 + 2c_1c_2) \times (a + c_2)}$$

We can take $(a + c_2)$ as a common factor to rewrite the previous expression as:

$$D(Q) = \frac{b \times \Big((a + c_2) \times \big(4 \times (a^2 + 2ac_1 + 2ac_2 + 2c_1c_2) - 2a \times (a + 2c_2)\big) - a \times (a^2 + 2ac_2 + 4ac_1 + 4c_1c_2)\Big)}{4 \times (a^2 + 2ac_1 + 2ac_2 + 2c_1c_2) \times (a + c_2)}$$

If we expand the term $2a \times (a + 2c_2)$, we get:

$$D(Q) = \frac{b \times \left((a + c_2) \times (4 \times (a^2 + 2ac_1 + 2ac_2 + 2c_1c_2) - 2a^2 - 4ac_2) - a \times (a^2 + 2ac_2 + 4ac_1 + 4c_1c_2)\right)}{4 \times (a^2 + 2ac_1 + 2ac_2 + 2c_1c_2) \times (a + c_2)}$$

By expanding the term $4 \times (a^2 + 2ac_1 + 2ac_2 + 2c_1c_2)$, we get:

$$D(Q) = \frac{b \times \left((a + c_2) \times (4a^2 + 8ac_1 + 8ac_2 + 8c_1c_2 - 2a^2 - 4ac_2) - a \times (a^2 + 2ac_2 + 4ac_1 + 4c_1c_2)\right)}{4 \times (a^2 + 2ac_1 + 2ac_2 + 2c_1c_2) \times (a + c_2)}$$

Which can be simplified to:

$$D(Q) = \frac{b \times \left((a + c_2) \times (2a^2 + 8ac_1 + 4ac_2 + 8c_1c_2) - a \times (a^2 + 2ac_2 + 4ac_1 + 4c_1c_2)\right)}{4 \times (a^2 + 2ac_1 + 2ac_2 + 2c_1c_2) \times (a + c_2)}$$

By expanding $(a + c_2) \times (2a^2 + 8ac_1 + 4ac_2 + 8c_1c_2)$ into two parentheses, we get:

$$D(Q) = \frac{b \times \left(a \times (2a^2 + 8ac_1 + 4ac_2 + 8c_1c_2 -) + c_2 \times (2a^2 + 8ac_1 + 4ac_2 + 8c_1c_2) - a \times (a^2 + 2ac_2 + 4ac_1 + 4c_1c_2)\right)}{4 \times (a^2 + 2ac_1 + 2ac_2 + 2c_1c_2) \times (a + c_2)}$$

Noticing that we can take a as a common factor, we can rewrite the previous expression as:

$$D(Q) = \frac{b \times \left(a \times (2a^2 + 8ac_1 + 4ac_2 + 8c_1c_2 - a^2 - 2ac_2 - 4ac_1 - 4c_1c_2) + c_2 \times (2a^2 + 8ac_1 + 4ac_2 + 8c_1c_2)\right)}{4 \times (a^2 + 2ac_1 + 2ac_2 + 2c_1c_2) \times (a + c_2)}$$

And simplifying again, we get:

$$D(Q) = \frac{b \times \left(a \times (a^2 + 4ac_1 + 2ac_2 + 4c_1c_2) + c_2 \times (2a^2 + 8ac_1 + 4ac_2 + 8c_1c_2)\right)}{4 \times (a^2 + 2ac_1 + 2ac_2 + 2c_1c_2) \times (a + c_2)}$$

By taking 2 as a common factor in the expression $2a^2 + 8ac_1 + 4ac_2 + 8c_1c_2$ of the numerator, we get:

$$D(Q) = \frac{b \times \left(a \times (a^2 + 4ac_1 + 2ac_2 + 4c_1c_2) + 2c_2 \times (a^2 + 4ac_1 + 2ac_2 + 4c_1c_2)\right)}{4 \times (a^2 + 2ac_1 + 2ac_2 + 2c_1c_2) \times (a + c_2)}$$

Finally, taking $a^2 + 4ac_1 + 2ac_2 + 4c_1c_2$ as a common factor, we get:

$$D(Q) = \frac{b \times \left((a + 2c_2) \times (a^2 + 4ac_1 + 2ac_2 + 4c_1c_2)\right)}{4 \times (a^2 + 2ac_1 + 2ac_2 + 2c_1c_2) \times (a + c_2)} = p^*$$

Notice that we can rewrite this expression in terms of q_1^* and q_2^*, which are given by:

$$\frac{b \times (a^2 + 2ac_2 + 4ac_1 + 4c_1c_2)}{4 \times (a^2 + 2ac_1 + 2ac_2 + 2c_1c_2) \times (a + c_2)} = q_2^*$$

$$\frac{b \times (a + 2c_2)}{2 \times (a^2 + 2ac_1 + 2ac_2 + 2c_1c_2)} = q_1^*$$

Hence, it trivially follows that:

$$(a^2 + 2ac_2 + 4ac_1 + 4c_1c_2) = q_2^* \times \frac{4 \times (a^2 + 2ac_1 + 2ac_2 + 2c_1c_2) \times (a + c_2)}{b}$$

$$(a + 2c_2) = q_1^* \times \frac{2 \times (a^2 + 2ac_1 + 2ac_2 + 2c_1c_2)}{b}$$

Substituting $(a^2 + 2ac_2 + 4ac_1 + 4c_1c_2)$ and $(a + 2c_2)$ into p^*, we get:

$$p^* = \frac{b \times \left(q_1^* \times \frac{2 \times (a^2 + 2ac_1 + 2ac_2 + 2c_1c_2)}{b} \times q_2^* \times \frac{4 \times (a^2 + 2ac_1 + 2ac_2 + 2c_1c_2) \times (a + c_2)}{b}\right)}{4 \times (a^2 + 2ac_1 + 2ac_2 + 2c_1c_2) \times (a + c_2)}$$

Which can be rewritten as:

$$p^* = \frac{b \times \left(\frac{q_1^* \times q_2^* \times 2 \times (a^2 + 2ac_1 + 2ac_2 + 2c_1c_2) \times 4 \times (a^2 + 2ac_1 + 2ac_2 + 2c_1c_2) \times (a + c_2)}{b^2}\right)}{4 \times (a^2 + 2ac_1 + 2ac_2 + 2c_1c_2) \times (a + c_2)}$$

Noting that $\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{ad}{bc}$, we can rewrite the previous expression as:

$$p^* = b \times \left(\frac{q_1^* \times q_2^* \times 2 \times (a^2 + 2ac_1 + 2ac_2 + 2c_1c_2) \times 4 \times (a^2 + 2ac_1 + 2ac_2 + 2c_1c_2) \times (a + c_2)}{b^2 \times 4 \times (a^2 + 2ac_1 + 2ac_2 + 2c_1c_2) \times (a + c_2)}\right)$$

And, simplifying, we get:

$$p^* = \frac{q_1^* \times q_2^* \times 2 \times (a^2 + 2ac_1 + 2ac_2 + 2c_1c_2)}{b}$$

We can write the profit of firms 1 and 2 as:

$$\pi_1(q_1, q_2) = p^* \times q_1^* - c_1 q_1^{*^2}$$

$$\pi_2(q_1, q_2) = p^* \times q_2^* - c_1 q_2^{*^2}$$

Which, after substituting p^*, q_1^* and q_2^*, the profit of firm 1 becomes:

$$\pi_1(q_1, q_2) = q_1^{*^2} \times \frac{q_2^* \times 2 \times (a^2 + 2ac_1 + 2ac_2 + 2c_1c_2)}{b} - c_1 q_1^2$$

$$\pi_1(q_1, q_2) = q_1^2 \times \left(\frac{q_2^* \times 2 \times (a^2 + 2ac_1 + 2ac_2 + 2c_1c_2)}{b} - c_1\right)$$

$$\pi_1(q_1, q_2) = q_1^2 \times \left(\frac{\frac{b \times \left(a \times (a + 2c_2) + 4c_1 \times (a + c_2)\right)}{4 \times (a^2 + 2ac_1 + 2ac_2 + 2c_1c_2) \times (a + c_2)} \times 2 \times \left((a^2 + 2ac_1 + 2ac_2 + 2c_1c_2)\right)}{b} - c_1\right)$$

Which can be rewritten as:

$$\pi_1(q_1, q_2) = q_1^2 \times \left(\frac{b \times \left(a \times (a + 2c_2) + 4c_1 \times (a + c_2)\right) \times 2 \times \left((a^2 + 2ac_1 + 2ac_2 + 2c_1c_2)\right)}{4 \times (a^2 + 2ac_1 + 2ac_2 + 2c_1c_2) \times (a + c_2) \times b} - c_1\right)$$

And, simplifying, we get:

$$\pi_1(q_1, q_2) = q_1^2 \times \left(\frac{\left(a \times (a + 2c_2) + 4c_1 \times (a + c_2)\right)}{2 \times (a + c_2)} - c_1\right)$$

Which can be rewritten as:

$$\pi_1(q_1, q_2) = q_1^2 \times \left(\frac{a^2 + 2ac_2 + 4ac_1 + 4c_1c_2}{2 \times (a + c_2)} - c_1\right)$$

Multiplying c_1 by $\frac{2\times(a+c_2)}{2\times(a+c_2)} = 1$, we get:

$$\pi_1(q_1,q_2) = q_1^2 \times \left(\frac{a^2 + 2ac_2 + 4ac_1 + 4c_1c_2 - c_1 \times 2 \times (a + c_2)}{2 \times (a + c_2)}\right)$$

Expanding the second parenthesis in the numerator, we get:

$$\pi_1(q_1,q_2) = q_1^2 \times \left(\frac{a^2 + 2ac_2 + 4ac_1 + 4c_1c_2 - 2ac_1 - 2c_1c_2}{2 \times (a + c_2)}\right)$$

Which, by simplifying, gets:

$$\pi_1(q_1,q_2) = q_1^2 \times \left(\frac{a^2 + 2ac_2 + 2ac_1 + 2c_1c_2}{2 \times (a + c_2)}\right)$$

Which can be rewritten as:

$$\pi_1(q_1,q_2) = q_1^2 \times \left(\frac{a^2 + 2ac_2 + 2ac_1 + 2c_1c_2}{2 \times (a + c_2)}\right)$$

Analogously, after substituting p^*, q_1^* and q_2^*, the profit of firm 1 becomes:

$$\pi_2(q_1,q_2) = {q_2^*}^2 \times \frac{q_1^* \times 2 \times (a^2 + 2ac_1 + 2ac_2 + 2c_1c_2)}{b} - c_2q_2^2$$

$$\pi_2(q_1,q_2) = q_2^2 \times \left(\frac{q_1^* \times 2 \times (a^2 + 2ac_1 + 2ac_2 + 2c_1c_2)}{b} - c_2\right)$$

$$\pi_2(q_1,q_2) = q_2^2 \times \left(\frac{\frac{b \times (a + 2c_2)}{2 \times (a^2 + 2ac_1 + 2ac_2 + 2c_1c_2)} \times 2 \times \left((a^2 + 2ac_1 + 2ac_2 + 2c_1c_2)\right)}{b} - c_2\right)$$

Which can be rewritten as:

$$\pi_2(q_1,q_2) = q_2^2 \times \left(\frac{b \times (a + 2c_2) \times 2 \times \left((a^2 + 2ac_1 + 2ac_2 + 2c_1c_2)\right)}{2 \times (a^2 + 2ac_1 + 2ac_2 + 2c_1c_2) \times b} - c_2\right)$$

And, simplifying, we get:

$$\pi_2(q_1, q_2) = q_2^2 \times (a + 2c_2 - c_2)$$

Hence,

$$\pi_2(q_1, q_2) = q_2^2 \times (a + c_2)$$